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Closed-Form Analytical Solution for the Shimmy Instability of Nose-Wheel Landing Gears

Niranjan K. Sura*
Aeronautical Development Agency, Bangalore 560 017, India
and
S. Suryanarayan†
Indian Institute of Technology, Bombay, Mumbai 400 076, India

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Closed-form analytical expressions for the shimmy velocity and shimmy frequency are obtained, considering the lateral dynamics of a three-degree-of-freedom nose-wheel landing gear shimmy model. The analytical solution is based on the observation that the lowest modal frequency of the nose-wheel landing gear on the ground gives a close approximation to the shimmy frequency. Results are obtained for the shimmy frequency and shimmy velocity using these analytical expressions for a range of values of landing gear parameters and compared with the results obtained using an exact solution of the more general formulation. These results show that the analytical expressions can be used as a good first-cut approximation of the critical stability values for use at the early design stage.

Nomenclature

C_S = equivalent structural damping coefficient of the strut in lateral bending
$\bar{C}_S$ = nondimensional structural lateral damping coefficient of the strut
C_{Sh} = equivalent viscous damping coefficient in torsion (from the shimmy damper)
$\bar{C}_{\mathrm{Sh}}$ = nondimensional viscous damping coefficient in torsion
C_Δ = lateral damping of the tire
$\bar{C}_\Delta$ = nondimensional lateral damping of the tire
C_θ = equivalent structural damping coefficient of the strut in torsion
$\bar{C}_\theta$ = nondimensional structural damping coefficient of the strut in torsion
F_N = side force due to lateral flexibility and damping of the tire
K_S = lateral stiffness of the landing gear strut
K_Δ = lateral stiffness of the tire
$\bar{K}_\Delta$ = nondimensional lateral stiffness of the tire
K_θ = torsional stiffness of the landing gear strut
I = moment of inertia of the wheel-strut assembly about the gear vertical axis
$\bar{I}$ = nondimensional moment of inertia
L = distance of the axis of the wheel rotation from the gear vertical axis (caster length)
$\bar{L}$ = ratio of the caster length to the distance of the center of gravity of the wheel assembly from the gear vertical axis
L_{cg} = distance of the center of gravity of the wheel assembly from the gear vertical axis
m = mass of the wheel assembly
s = Laplace variable
$\bar{s}$ = nondimensional Laplace variable
t = dimensional time
V = landing gear forward velocity
$\bar{V}$ = nondimensional velocity
V_{ts} = tire contact point velocity (velocity of tire slip)
y = lateral displacement of the strut
$\bar{y}$ = nondimensional lateral displacement of the strut
Δ = lateral displacement of the tire
$\bar{\Delta}$ = nondimensional lateral displacement of the tire
θ = rotation of the wheel about the gear vertical axis
$\sigma, -\sigma$ = real part of s, modal damping
τ = nondimensional time
Ω = ratio of landing gear strut torsional frequency to lateral frequency
ω = modal circular frequency
$\bar{\omega}$ = nondimensional modal circular frequency
ω_S = uncoupled landing gear strut lateral circular frequency
ω_θ = uncoupled landing gear strut torsional circular frequency
ω_{1g}, ω_{2g} = coupled landing gear strut (resting on the ground) circular frequencies

I. Introduction

THE dynamics of nose-wheel landing gear (NLG) systems consists of 1) uncoupled symmetric longitudinal dynamics involving fore and aft and up and down motions and rotation about the lateral axis at the root of the landing gear strut and 2) uncoupled lateral dynamics that involve lateral motion, rotation of the wheel assembly about the fore and aft axis at the root of the strut, swiveling of the wheel assembly, and lateral and yaw deformations of the tire. Shimmy of the NLG is a self-excited dynamic lateral instability that may occur during takeoff, taxiing, and landing, involving mainly three vibratory motions: lateral displacement of the strut, rotation of the wheel assembly about the vertical axis (yaw), and rotation about the fore and aft axis (roll). Over the years, NLG shimmy has been studied by a number of authors [1–9]. However, studies covering the effect of various problem parameters on the onset of shimmy have been limited [1–6]. For such studies, simplified analytical models that capture the basic system characteristics are very suitable. Reference [1] presents studies on such a simplified three-degree-of-freedom (3-DOF) NLG model. In this reference, a linear NLG model with single-wheel configuration was considered, accounting for structural inertia, stiffness, and damping. Further, it was assumed that the tire is only laterally flexible, the interaction between the ground and the tire occurs at a single point, and the tire does not skid with respect to the ground. In this study, the force-deflection characteristics of the tire

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*Scientist, National Control Law Team, Integrated Flight Control Systems Directorate.
†Professor (Retired), Aerospace Engineering Department.

was based on Moreland's point-contact model [5–9] and was represented by a linear mathematical relationship expressing the side force acting on the wheel as a linear function of the lateral tire deflection and the rate of its change with time.

This paper presents closed-form analytical solutions for the shimmy frequency and shimmy velocity for the 3-DOF linear NLG model considered in [1]. An analysis of the results presented in [1] shows that the shimmy instability occurs at a frequency (shimmy frequency) very close to the lowest natural frequency of the NLG resting on the ground. The tire in contact with the ground provides effective additional stiffness for the lateral and torsional motions of the NLG. Based on this observation, using the closed-form analytical expression for the lowest natural frequency of the NLG on the ground, it is shown that the stability formulation for the 3-DOF linear NLG model yields a closed-form analytical expression for the shimmy velocity.

II. Analytical Formulation

Figures 1a and 1b, respectively, show a typical NLG configuration [10] and a schematic representation of the same as a cantilever supported from the fuselage. The landing gear flexibility may cause fore and aft motion in the x direction and lateral motion in the y direction. The vertical motion in the z direction is absorbed by the oleopneumatic shock absorber. Landing gear may also rotate about the fore and aft axis (x axis) because of lateral bending. The wheel is free to swivel about the vertical axis when the steering is not engaged. This DOF helps with steering the aircraft. The steering moment is transferred to the wheels using a rack-and-pinion mechanism through a torque link. The steering is assumed to be hydraulically controlled and incorporated with two spring-loaded hydraulic steering cylinders that serve as a steering mechanism and are also used to subdue torsional oscillations automatically. The damping is accomplished by the metering of hydraulic fluid through a small orifice between two cylinders. In the present study, the NLG is assumed to be equipped with such a damping mechanism. The schematic of such a steering mechanism is shown in Fig. 1c. The NLG model considered for the present paper accounts for structural inertia, stiffness, and structural damping. The torsional stiffness of the strut is the resultant of stiffness offered by the torque link, the hydraulic spring in the steering actuator, and the structure above the torque link, including the retraction jack. The damping of the system is assumed to result from the inherent structural damping in lateral and torsional DOF and external viscous damping in torsional DOF coming from the steering damping mechanism. The landing gear nonlinearities such as nonlinear lateral strut flexibility, friction in the oleostrut, free play in the steering, and nonlinear tire behavior are ignored.

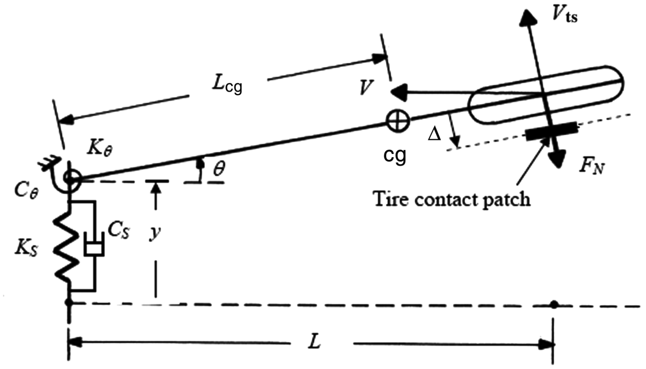


Fig. 2 Three-degree-of-freedom NLG shimmy model.

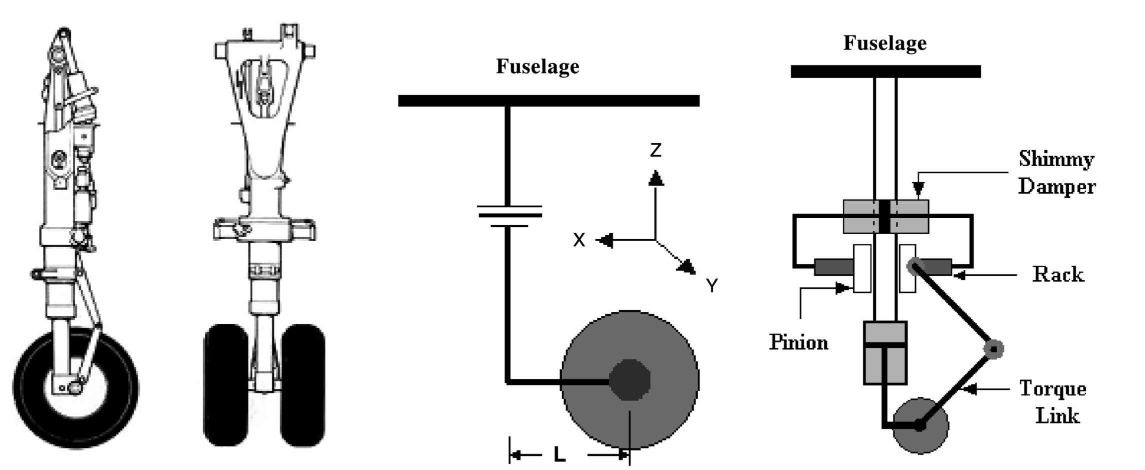
Figure 2 shows the schematic of the 3-DOF NLG model with lateral displacement of the wheel y, angular wheel motion (yaw) about the vertical axis θ, and lateral deflection of the tire contact patch with respect to the wheel center plane Δ as degrees of freedom [1]. It is assumed that the mass of the swivel wheel assembly m is lumped at the center of gravity (cg) of the wheel assembly. Let I be the moment of inertia of the wheel-strut assembly about the gear vertical axis, let L be the distance of the axis of the wheel rotation from the gear vertical axis (caster length), and let L_{cg} be the distance of the center of gravity of the wheel assembly from the gear vertical axis. Let K_S and C_S, respectively, represent the stiffness and equivalent structural damping coefficient of the NLG strut in lateral bending, let K_θ and C_θ represent the same in torsion, and let K_Δ and C_Δ represent lateral stiffness and lateral damping coefficient of the tire, respectively. Let C_{Sh} be the coefficient of equivalent viscous damping in torsion provided by an external means (the shimmy damper), in addition to the structural damping C_θ.

Let F_N be the side force acting on the wheel due to tire lateral deformation Δ. Assuming θ to be small ($\cos\theta = 1$ and $\sin\theta = \theta$) and that there is no tire slippage with respect to the ground, the equations of motion [1] for the 3-DOF NLG model can be written as

$$m\frac{d^2y}{dt^2} + mL_{cg}\frac{d^2\theta}{dt^2} + C_S\frac{dy}{dt} + K_S y + F_N = 0 \tag{1}$$

$$I\frac{d^2\theta}{dt^2} + mL_{cg}\frac{d^2y}{dt^2} + (C_\theta + C_{Sh})\frac{d\theta}{dt} + K_\theta\theta + F_N L = 0 \tag{2}$$

$$V\theta + \frac{dy}{dt} + L\frac{d\theta}{dt} - \frac{d\Delta}{dt} = 0 \tag{3}$$



a) Typical aircraft NLG

b) Schematic of NLG showing torsional DOF

c) Rack-and-pinion steering mechanism with shimmy damper

Fig. 1 Details of the NLG components and its schematic representation.

Equation (3) represents the kinematic condition that represents zero tire lateral slip with respect to the ground (i.e., $V_{ts} = 0$). According to Moreland's point-contact model, the interaction between the ground and the tire could be treated as occurring at a single point, and the elastic restoring effect of the tire on the wheel (i.e., side force F_N) is assumed to be linearly proportional to the lateral deflection of the tire Δ and to the rate of its change with time $\mathrm{d}\Delta/\mathrm{d}t$. Thus, F_N is given by

$$F_N = K_\Delta \Delta + C_\Delta \frac{\mathrm{d}\Delta}{\mathrm{d}t} \tag{4}$$

Defining $\omega_S^2 = (K_S/m)$, $\omega_\theta^2 = (K_\theta/I)$, and the following nondimensional parameters,

$$\bar{y} = \frac{y}{L}, \quad \bar{\Delta} = \frac{\Delta}{L}, \quad \bar{L} = \frac{L_{cg}}{L}, \quad \bar{I} = \frac{I}{mL^2}, \quad \tau = t\omega_S$$
$$\Omega = \frac{\omega_\theta}{\omega_S}, \quad \bar{V} = \frac{V}{\omega_S L} \quad \bar{K}_\Delta = \frac{K_\Delta}{m\omega_S^2}, \quad \bar{C}_\Delta = \frac{C_\Delta}{m\omega_S}$$
$$\bar{C}_S = \frac{C_S}{m\omega_S}, \quad \bar{C}_\theta = \frac{C_\theta}{mL^2\omega_S}, \quad \bar{C}_{Sh} = \frac{C_{Sh}}{mL^2\omega_S} \tag{5}$$

Equations (1–4) can be written in a nondimensional form as

$$\ddot{\bar{y}} + \bar{L}\,\ddot{\theta} + \bar{C}_S\dot{\bar{y}} + \bar{y} + \bar{K}_\Delta\bar{\Delta} + \bar{C}_\Delta\dot{\bar{\Delta}} = 0 \tag{6}$$

$$\bar{L}\,\ddot{\bar{y}} + \bar{I}\,\ddot{\theta} + (\bar{C}_\theta + \bar{C}_{Sh})\dot{\theta} + \bar{I}\Omega^2\theta + \bar{K}_\Delta\bar{\Delta} + \bar{C}_\Delta\dot{\bar{\Delta}} = 0 \tag{7}$$

$$\dot{\bar{y}} + \dot{\theta} + \bar{V}\theta - \dot{\bar{\Delta}} = 0 \tag{8}$$

In the preceding equations, single and double dots over the quantities represent the first and second derivatives with respect to the nondimensional time parameter τ. Equations (6–8) represent the dynamics of the 3-DOF system in a nondimensional form. The stability of the system can be studied by solving for complex eigenvalues of the system computed at various velocities. Equations (6–8) can be written in the Laplace domain in terms of nondimensional Laplace variable $\bar{s}$ as

$$(\bar{s}^2 + \bar{s}\bar{C}_S + 1)\bar{y} + \bar{s}^2\bar{L}\theta + (\bar{K}_\Delta + \bar{s}\bar{C}_\Delta)\bar{\Delta} = 0 \tag{9}$$

$$\bar{s}^2\bar{L}\,\bar{y} + (\bar{s}^2\bar{I} + \bar{s}\bar{C}_\theta + \bar{s}\bar{C}_{Sh} + \bar{I}\Omega^2)\theta + (\bar{K}_\Delta + \bar{s}\bar{C}_\Delta)\bar{\Delta} = 0 \tag{10}$$

$$\bar{s}(\bar{y} + \theta - \bar{\Delta}) + \bar{V}\theta = 0 \tag{11}$$

We can get the characteristic equation by substituting $\bar{\Delta}$ from Eq. (11) into Eqs. (9) and (10) as

$$\left[(\bar{s}^2 + \bar{s}\bar{C}_S + 1) + \bar{s}^2\bar{L}R_{\theta\bar{y}} + (\bar{K}_\Delta + \bar{s}\bar{C}_\Delta)\left(1 + R_{\theta\bar{y}} + \bar{V}\frac{R_{\theta\bar{y}}}{\bar{s}}\right)\right]\bar{y} = 0 \tag{12}$$

where

$$R_{\theta\bar{y}} = \frac{(\bar{s}^2 + \bar{s}\bar{C}_S + 1 - \bar{s}^2\bar{L})}{(\bar{s}^2\bar{I} + \bar{s}\bar{C}_\theta + \bar{s}\bar{C}_{Sh} + \bar{I}\Omega^2 - \bar{s}^2\bar{L})} \tag{13}$$

By substituting Eq. (13) in the preceding characteristic equation and solving it for complex eigenvalues, we get five characteristic roots as

$$\bar{\lambda}_{1,2} = \bar{\sigma}_1 \pm i\bar{\omega}_1, \qquad \bar{\lambda}_{3,4} = \bar{\sigma}_2 \pm i\bar{\omega}_2, \qquad \bar{\lambda}_5 = \bar{\sigma}_3 \tag{14}$$

where $\bar{\sigma}_1$, $\bar{\sigma}_2$, and $\bar{\sigma}_3$ represent modal damping of the natural modes of the system, and $\bar{\omega}_1$ and $\bar{\omega}_2$ represent the modal frequencies. At the point of instability (critical condition), one of the real part $\bar{\sigma}_{1,2}$ of the preceding complex conjugate roots becomes zero and we have $\bar{\lambda} = i\bar{\omega}_{Cr}$, where $\bar{\omega}_{Cr}$ is the critical shimmy frequency. At the critical condition, because the value of $\bar{\sigma} = 0$ and $\bar{s} = i\bar{\omega}_{Cr}$, the system will have a steady-state response. Substituting for $\bar{s} = i\bar{\omega}_{Cr}$ in Eq. (12), a closed-form expression for shimmy velocity $\bar{V}_{Cr}$ can be obtained in terms of shimmy frequency $\bar{\omega}_{Cr}$ as

$$\bar{V}_{Cr} = i\bar{\omega}_{Cr}\frac{(1 - \bar{\omega}_{Cr}^2 + i\bar{\omega}_{Cr}\bar{C}_S) - \bar{\omega}_{Cr}^2\bar{L}R_{\theta\bar{y}} + (\bar{K}_\Delta + i\bar{\omega}_{Cr}\bar{C}_\Delta)(1 + R_{\theta\bar{y}})}{(\bar{K}_\Delta + i\bar{\omega}_{Cr}\bar{C}_\Delta)R_{\theta\bar{y}}} \tag{15}$$

where

$$R_{\theta\bar{y}} = \frac{[(\bar{L} - 1)\bar{\omega}_{Cr}^2 + i\bar{C}_S\bar{\omega}_{Cr} + 1]}{[(\bar{L} - \bar{I})\bar{\omega}_{Cr}^2 + i(\bar{C}_\theta + \bar{C}_{Sh})\bar{\omega}_{Cr} + \bar{I}\Omega^2]} \tag{16}$$

A preliminary analysis of the numerical results obtained for the shimmy instability of the NLG model showed that shimmy occurs at a frequency slightly higher than the lowest natural frequency of the NLG off the ground. Considering the NLG resting on the ground and accounting for the contribution of the tire to lateral stiffness and damping, it is seen that the shimmy frequency is very close to the lowest natural frequency of the NLG on the ground. Using the explicit analytical expression for the lowest natural frequency of the NLG system on the ground for the value of the shimmy frequency in Eq. (15), we get an explicit analytical expression for shimmy velocity.

When the landing gear is resting on the ground without forward motion, setting $\bar{V} = 0$, the nondimensional equations of motion of the 3-DOF NLG model given by Eqs. (6–8) can be reduced to

$$\ddot{\bar{y}} + (\bar{C}_S + \bar{C}_\Delta)\dot{\bar{y}} + (1 + \bar{K}_\Delta)\bar{y} + \bar{L}\,\ddot{\theta} + \bar{C}_\Delta\dot{\theta} + \bar{K}_\Delta\theta = 0 \tag{17}$$

$$\bar{L}\,\ddot{\bar{y}} + \bar{C}_\Delta\dot{\bar{y}} + \bar{K}_\Delta\bar{y} + \bar{I}\,\ddot{\theta} + (\bar{C}_\theta + \bar{C}_{Sh} + \bar{C}_\Delta)\dot{\theta} + (\bar{I}\Omega^2 + \bar{K}_\Delta)\theta = 0 \tag{18}$$

For $\bar{V} = 0$, the no-slip condition given by Eq. (8) yields $\dot{\bar{\Delta}} = \dot{\bar{y}}$ and the system reduces to a 2-DOF system. For this case, the solution can be obtained as $\bar{y} = \bar{y}_0 e^{\lambda\tau}$ and $\theta = \theta_0 e^{\lambda\tau}$, and for the nontrivial solution of $\bar{y}$ and θ, we have

$$\left|\begin{bmatrix} \lambda^2 + (\bar{C}_S + \bar{C}_\Delta)\lambda + \bar{K}_\Delta + 1 & \lambda^2\bar{L} + \bar{C}_\Delta\lambda + \bar{K}_\Delta \\ \lambda^2\bar{L} + \bar{C}_\Delta\lambda + \bar{K}_\Delta & \lambda^2\bar{I} + (\bar{C}_\theta + \bar{C}_{Sh} + \bar{C}_\Delta)\lambda + \bar{I}\Omega^2 + \bar{K}_\Delta \end{bmatrix}\right| = 0 \tag{19}$$

The solutions of the preceding equation can be obtained [as in Eq. (14)] as

$$\bar{\lambda}_{1,2} = \bar{\sigma}_1 \pm i\bar{\omega}_1, \qquad \bar{\lambda}_{3,4} = \bar{\sigma}_2 \pm i\bar{\omega}_2 \tag{20}$$

where $\bar{\omega}_1$ and $\bar{\omega}_2$ represent damped natural frequencies of the two modes, and $\bar{\sigma}_1$ and $\bar{\sigma}_2$ represent the respective modal damping. It is also observed from [1] that for all values of C_{Sh}, the shimmy frequency is approximately equal to coupled lateral strut frequency and the shimmy frequency decreases marginally with an increase in viscous damping. If damping is ignored, the solution for Eqs. (17) and (18) can be obtained as $\bar{y} = \bar{y}_0 e^{i\bar{\omega}\tau}$ and $\theta = \theta_0 e^{i\bar{\omega}\tau}$, and for nontrivial solution of $\bar{y}$ and θ, we have

$$\left|\begin{bmatrix} -\bar{\omega}^2 + (1 + \bar{K}_\Delta) & -\bar{\omega}^2\bar{L} + \bar{K}_\Delta \\ -\bar{\omega}^2\bar{L} + \bar{K}_\Delta & -\bar{\omega}^2\bar{I} + \bar{I}\Omega^2 r + \bar{K}_\Delta \end{bmatrix}\right| = 0 \tag{21}$$

The solution of the preceding equation will yield the undamped natural frequencies and mode shapes of the 2-DOF coupled system. The roots of the preceding characteristic equation are given by

$$\bar{\omega}_{2,1} = \frac{\{[\bar{K}_\Delta(1 + \bar{I} - 2\bar{L})] + \bar{I}(1 + \Omega^2)\}}{2(\bar{I} - \bar{L}^2)} \pm \left(\frac{[\bar{K}_\Delta(1 + \bar{I} - 2\bar{L}) + \bar{I}(1 + \Omega^2)]^2}{4(\bar{I} - \bar{L}^2)^2} - \frac{[\bar{I}\Omega^2 + \bar{K}_\Delta(1 + \bar{I}\Omega^2)]}{(\bar{I} - \bar{L}^2)}\right)^{(1/2)} \tag{22}$$

It is observed [1] that the shimmy frequency of the NLG is very nearly equal to the lower of the two natural frequencies of the NLG system on the ground, the analytical expression for shimmy frequency $\bar{\omega}_{\mathrm{Cr}}$ (nondimensional) can be obtained from Eq. (22) as

$$\bar{\omega}_{\mathrm{Cr}} \approx \bar{\omega}_1 = \frac{\omega_{1g}}{2(1-\rho)\omega_S}\left\{(\Omega' + 1 + 2\sqrt{\rho\beta\Omega'}) - [(\Omega' + 1 + 2\sqrt{\rho\beta\Omega'})^2 - 4(1-\Omega')(1-\beta)]^{(1/2)}\right\} \tag{23}$$

where

$$\rho = \frac{mL_{\mathrm{cg}}^2}{I}, \qquad \beta = \frac{(K_\Delta L)^2}{(K_S + K_\Delta)(K_\theta + K_\Delta L^2)}, \qquad \Omega' = \left(\frac{\omega_{2g}}{\omega_{1g}}\right)^2$$

$$\omega_{1g}^2 = \frac{K_S + K_\Delta}{m}, \qquad \omega_{2g}^2 = \frac{K_\theta + K_\Delta L^2}{I} \tag{24}$$

Critical shimmy velocity of the preceding 3-DOF NLG model can be obtained by substituting the preceding analytical expression for critical shimmy frequency given by Eq. (23) into the analytical expression for critical shimmy velocity given by Eq. (15).

III. Results and Discussion

The studies presented here consider the baseline values of various problem parameters for a typical nose-wheel landing gear configuration of a fighter class of aircraft, as given in Table 1. The effect of variation of some of the problem parameters is also presented for a range of values, as given in Table 1.

Free-vibration analysis of the NLG model is carried out, setting forward velocity to zero for the baseline values of problem parameters. When the tire is not in contact with the ground, the uncoupled lateral and torsional frequencies obtained, ignoring inertia and stiffness coupling, are 25 and 75 Hz, respectively. When the effect of inertia and stiffness coupling is considered, these frequencies are, respectively, 29.62 and 93.88 Hz. For this case, it is observed that the shimmy frequency is very close to 29.62 Hz. Figure 3 shows modal frequencies for various values of torsional stiffness parameter Ω obtained using the analytical expression given by Eq. (22) for the baseline values of the NLG configuration compared with those of the shimmy frequency at the onset of instability obtained from the complex eigenvalue approach [1] (exact solution). For the entire range of Ω shown in Fig. 3, it can be seen that the lowest modal frequency obtained from the analytical expression matches closely with the shimmy frequency.

Table 1 Values of the NLG parameters for 3-DOF baseline problems and their ranges

Baseline values of NLG parameters	
Strut inertia parameters	$m = 22$ kg and $I_{\mathrm{cg}} = 0.198$ kgm^2
Strut geometric parameters	$L = 0.075$ m and $L_{\mathrm{cg}} = 0.0675$ m
Strut stiffness parameters	$K_S = 542.83$ kN/m and $\Omega = 3$
Strut damping parameters	$\bar{C}_S = 0.01$, $\bar{C}_\theta = 0.02$, and $\bar{C}_{\mathrm{Sh}} = 0$
Tire parameters	$K_\Delta = 238.75$ kN/m and $C_\Delta = 205$ Ns/m
Range of values of NLG parameters	
Forward velocity	$V = 0$–300 kmph
Shimmy damping	$C_{\mathrm{Sh}} = 0$–100 Nms/rad
Strut stiffness	$\Omega = 0$–3

Results were obtained for the shimmy frequency and shimmy velocity using the analytical expressions given by Eqs. (23) and (15), respectively, and compared with those obtained from the exact solution. Figures 4–7 show a comparison of variation of Ω with the shimmy velocity and shimmy frequency for the range of values of torsional stiffness parameter $\Omega = 0$–3 for the shimmy damping values $C_{\mathrm{Sh}} = 0$, 25, 50, and 100 Nms/rad, respectively. Figures 8 and 9 show similar comparison of the variation of the shimmy velocity with the shimmy damping parameter C_{Sh} for $\Omega = 2$ and $\Omega = 3$, respectively. It can be observed from these results that for the values of C_{Sh} up to 50 Nms/rad and for values of Ω up to 3, the error in the calculation of the shimmy velocity is less than 4% and the values are on the conservative side. For the case of $C_{\mathrm{Sh}} = 100$ (see Fig. 7) at $\Omega = 1.6$, 2% error in the shimmy frequency results in approximately 11% error in the shimmy velocity. Table 2 shows the shimmy velocity for various values of Ω and shimmy damping C_{Sh} (in terms of percentage of critical damping in torsion) obtained from the exact solution (ES) and analytical expression (AE), along with the percentage error in the results obtained from the analytical expression (PeAE). However, for most practical cases, when $\Omega > 2$ and C_{Sh} is in the range of 25 to 50 Nms/rad, the error in the shimmy velocity obtained from the analytical expression is less than 2%.

The analytical expression for the shimmy velocity represented by Eq. (15) yields, in general, complex values for the shimmy velocity when an arbitrary value is assigned to the shimmy frequency. If the exact value of the shimmy frequency is substituted into Eq. (15), the imaginary component will become zero and the expression gives the shimmy velocity as a real value. If the value of the shimmy frequency

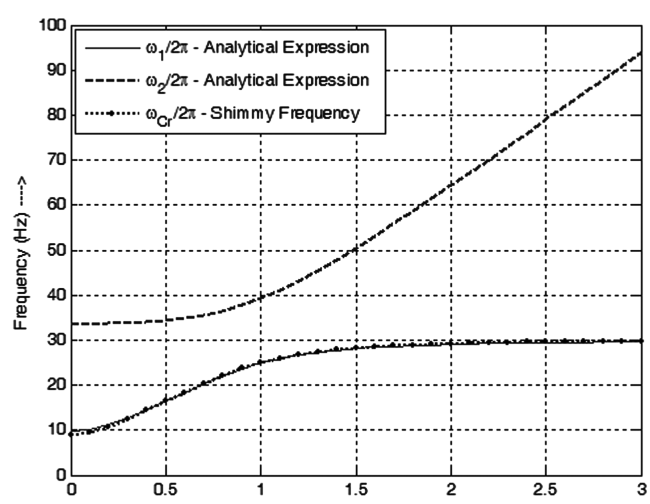


Fig. 3 Comparison of modal frequencies obtained using analytical expressions (at $V = 0$) with that of shimmy frequency obtained using an exact solution for baseline configuration.

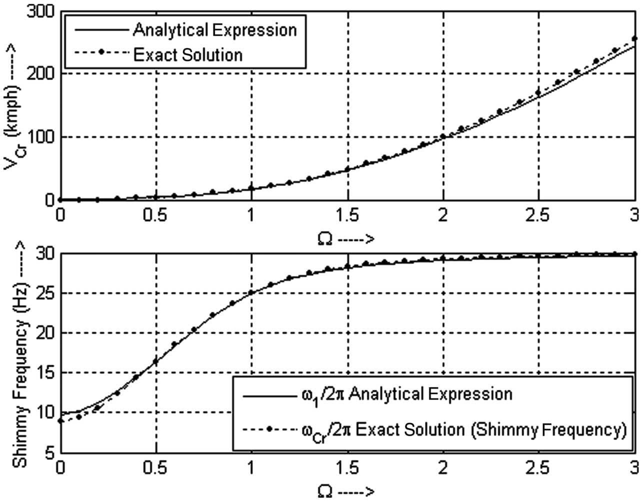


Fig. 4 Variation of V_{Cr} and shimmy frequency with Ω for $C_{\mathrm{Sh}} = 0$.

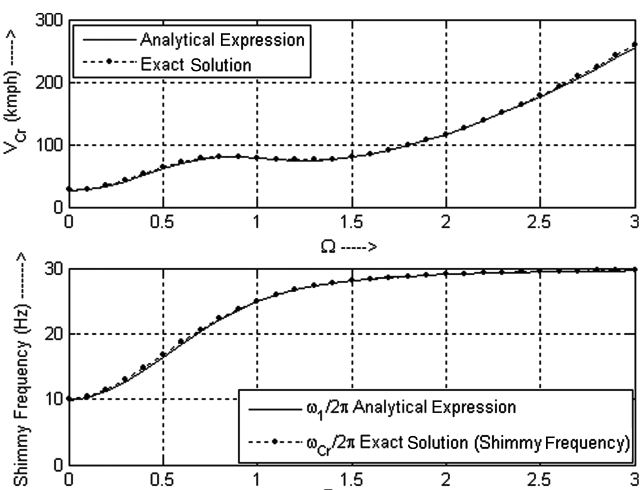


Fig. 5 Variation of V_{Cr} and shimmy frequency with Ω for $C_{\mathrm{Sh}} = 25\ \mathrm{Nms/rad}$.

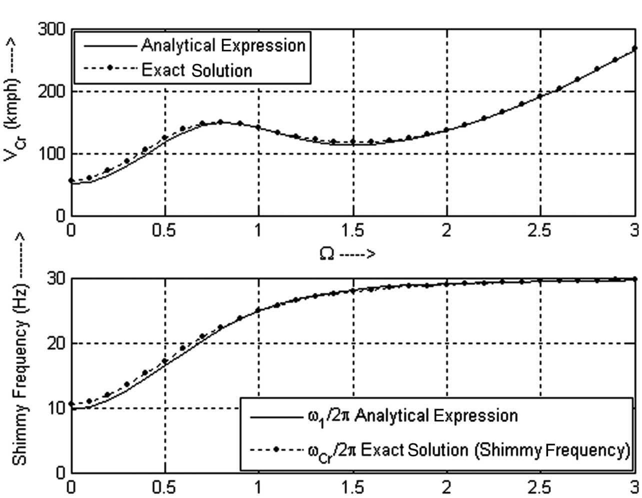


Fig. 6 Variation of V_{Cr} and shimmy frequency with Ω for $C_{\mathrm{Sh}} = 50\ \mathrm{Nms/rad}$.

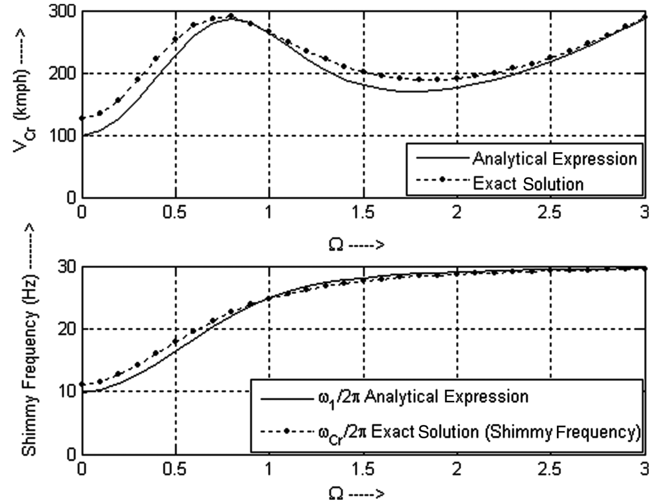


Fig. 7 Variation of V_{Cr} and shimmy frequency with Ω for $C_{\mathrm{Sh}} = 100\ \mathrm{Nms/rad}$.

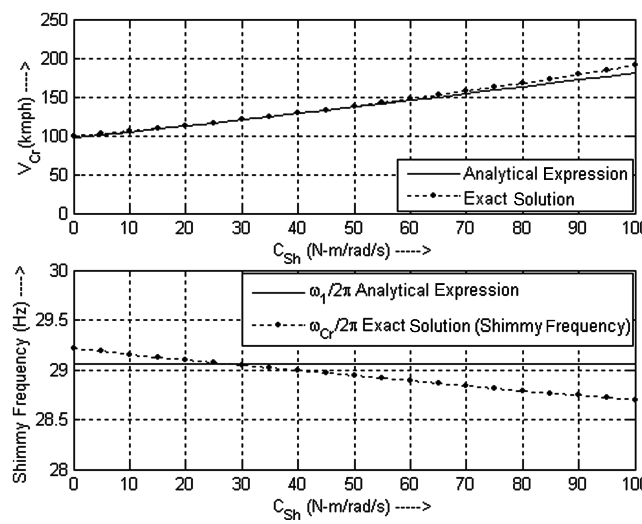


Fig. 8 Variation of V_{Cr} and shimmy frequency with shimmy damping parameter C_{Sh} for $\Omega = 2$.

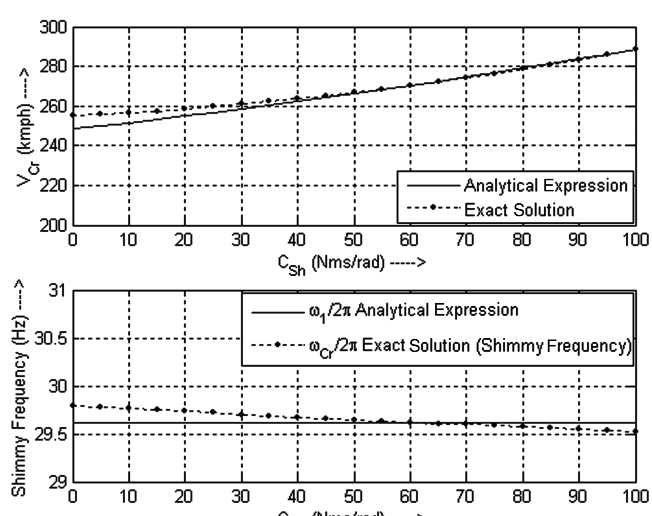


Fig. 9 Variation of V_{Cr} and shimmy frequency with shimmy damping parameter C_{Sh} for $\Omega = 3$.

Table 2 Comparison of shimmy velocity V_{Cr} (kmph) for various values of Ω and C_{Sh} obtained from the exact solution and analytical expression.

	% critical damping			$\Omega = 1$			$\Omega = 2$			$\Omega = 3$		
C_{Sh}, Nms/rad	$\Omega = 1$	$\Omega = 2$	$\Omega = 3$	ES	AE	PeAE	ES	AE	PeAE	ES	AE	PeAE
0	0	0	0	17.0	16.9	**0.60**	99.7	96.5	**3.21**	255.1	245.1	**3.92**
25	9	6	3	78.3	78.2	**0.14**	116.5	116.4	**0.08**	259.5	255.5	**1.54**
50	18	12	6	140.0	139.8	**0.14**	137.4	136.1	**0.95**	266.8	266.1	**0.26**
100	36	24	12	264.7	262.8	**0.72**	190	175	**8.00**	288.3	287.2	**0.38**

used is an approximate value, as given by Eq. (23), Eq. (15) yields the shimmy velocity with a small residual imaginary component on the order of 10^{-3} or lower in comparison with the real part. The accuracy of the shimmy velocity computed using the analytical expression depends on the input value of the shimmy frequency. When the experimental values are available for the natural frequencies of the NLG system on the ground, these values can be directly used for getting an estimate of the shimmy velocity.

IV. Conclusions

Using a simplified formulation for a 3-DOF linear NLG shimmy model, closed-form analytical expressions for the shimmy velocity and shimmy frequency were obtained. For all practical cases, when the torsional frequency of the NLG is more than two times the lateral frequency, the closed-form analytical expression gives an accurate estimate of the shimmy velocity, with an error of less than 2% when the lowest undamped natural frequency is used as the shimmy frequency. Hence, these closed-form analytical expressions obtained for the shimmy frequency and shimmy velocity can be used as a good first-cut approximation of the critical stability values at the early design stage.

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References

[1] Sura, N. K., and Suryanarayan, S., "Shimmy Studies on Simplified Linear Nose-Wheel Landing Gear Models," *Journal of Aerospace Sciences and Technologies*, Vol. 56, No. 1, Feb. 2004, pp. 49–54.

[2] Sura, N. K., and Suryanarayan, S., "Dynamic Response and Stability Studies on Simplified Models of Aircraft Landing Gears," *India-USA Symposium on Emerging Trends in Vibration and Noise Engineering* [CD-ROM], The Ohio State Univ., Columbus, OH, 10–12 Dec. 2001, Paper U048.

[3] Sura, N. K., and Suryanarayan, S., "Stability and Response Studies on Simplified Models of Nose-Wheel Landing Gear with Hard Tires," *Journal of the Institution of Engineers (India)*, Vol. 85, May 2004, pp. 29–36; also available online at http://www.ieindia.org/publish/as/0504/may04as5.pdf.

[4] Sura, N. K., Suryanarayan, S., Dipak K. M., and Upadhya, A. R., "Stability and Response Studies on Non-Linear Models of Nose-Wheel Landing Gears," *Proceedings of Aerospace and Related Mechanisms 2002*, Vikram Sarabhai Space Center, Trivendrum, Kerala, India, 8–9 Nov. 2002.

[5] Sura, N. K., and Suryanarayan, S., "Nonlinear Lateral Dynamics of Nose Wheel Landing Gears," *International Conference on Computational and Experimental Engineering and Sciences (ICCES)*, Indian Inst. of Technology Madras, Chennai, India, 1–6 Dec. 2005, Paper 0520051116053.

[6] Krabacher, W. E., "A Review of Aircraft Landing Gear Dynamics," AGARD Rept. R-800, 1996.

[7] Collins, R. L., "Theories on the Mechanics of Tires and Their Applications to Shimmy Analysis," *Journal of Aircraft*, Vol. 8, No. 4, 1971, pp. 271–277.

[8] Collins, R. L., "Frequency Response of Tires Using the Point Contact Theory," *Journal of Aircraft*, Vol. 9, No. 6, 1972, pp. 427–432.

[9] Smiley, R. F., "Correlation, Evaluation, and Extension of Linearized Theories for Tire Motion and Wheel Shimmy," NACA TM-1299, 1956.

[10] Young, D. W., "Aircraft Landing Gears—The Past Present and Future," Society of Automotive Engineers, Paper 864752, July 1985.

[11] Sura, N. K., "Lateral Stability and Response of Nose Wheel Landing Gears," Ph.D. Dissertation, Aerospace Engineering Dept., Indian Inst. of Technology, Bombay, Mumbai, India, Sept. 2004.